

Name: _____

Score: _____

1. Sketch the graphs of the following functions. (4 points each)

(a) $f(x) = x^4 - 2x^2 + 3$

(b) $f(x) = \frac{x^2}{x-1}$

2. (a) Show that $\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{2x} = f'(0)$ for every function $f(x)$. (Hint: l'Hopital's rule) (1 point)
- (b) Show that $\lim_{x \rightarrow 0} \frac{f(x) - 2f(0) + f(-x)}{x^2} = f''(0)$ for every function $f(x)$. (1 point)
- (c) Find numbers a, b, c, d such that $\lim_{x \rightarrow 0} \frac{af(3x) + bf(x) + cf(-x) + df(-3x)}{x^3} = f'''(0)$ for every function $f(x)$. (Bonus 1 point)